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AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
L. G. HANSCOM FIELD, BEDFORD, MASSACHUSETTS

Single Tee and Pi Two-Ports, Resistively Terminated and Having a Prescribed Driving-Point Immittance

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Abstract

Biquadratic, biquartic, bisextic, in general terms bi-order-n positive real immittance functions can be realized as driving-point immittances of a Tee or a Pi section that is terminated with a resistance, provided that the function belongs to a certain subclass of positive real and bi-order-n functions. This general principle is discussed particularly for biquadratic and biquartic functions. It is shown that the Tee and the Pi circuits implying a negative immittance of the rank 2m+1 can be transformed into another circuit that instead of the negative immittance implies a resistance as the only negative branch element.

illustrations

Single Tee and Pi Two-Ports, Resistively Terminated and Having a Prescribed Driving-Point Immittance

Consider the circuits shown in Figures 1 and 2. They are dual circuits when U, V, W, X, and Z in Figure 1 are branch impedances and in Figure 2, branch admittances. They have the driving-point immittance function

$$\bar{F}(s) = U + \cfrac{1}{\cfrac{1}{V+X} + \cfrac{1}{V+Z}}$$

$$= \frac{(UV + UW + VW) + (U + W)X + (U + V)Z + XZ}{V + W + X + Z} \quad . \qquad (1)$$

Both circuits in Figures 1 and 2 are resistively terminated and owing to their duality, it will suffice to restrict our discussion to the circuit in Figure 1. All results to be obtained will hold respectively for the circuit in Figure 2.

Let the branch impedances of the circuit in Figure 1 be

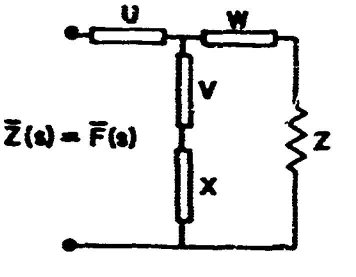

Figure 1. Circuit With the Impedance Function $\bar{Z}(s) = \bar{F}(s)$

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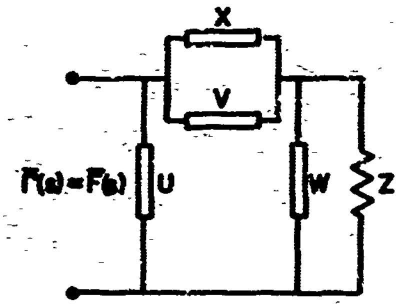

Figure 2. Circuit With Admittance Function $\Gamma(s) = F(s)$

$$U = u\varphi(s) \quad , \tag{2a}$$

$$V = v\varphi(s) \quad , \tag{2b}$$

$$W = w\varphi(s) \quad , \tag{2c}$$

$$X = x\Phi(s) \quad , \tag{3}$$

$$Z = z \quad . \tag{4}$$

In Eqs. (2a, b, c) let u and v be positive constants and let w be such that

$$1/u + 1/v + 1/w = 0 \text{ and therefore also } UV + UW + VW = 0 \tag{5}$$

Then with $n > 1$ and positive

$$u = v(n - 1) \, , \tag{6a}$$

$$w = - v(n - 1)/n \quad , \tag{6b}$$

Eq. (5) holds. Let x in Eq. (3) and z in Eq. (4) be also positive constants. Let $\varphi(s)$ in Eqs. (2a, b, c) be a normalized positive real (pr) function of the kind

$$\varphi(s) = s\,\frac{a(s)}{b(s)} = s\,\frac{s^m + a_{m-1}s^{m-1} + \ldots + a_1 s + a_0}{s^m + b_{m-1}s^{m-1} + \ldots + b_1 s + b_0} \quad . \tag{7}$$

Let $\Phi(s)$ in Eq. (3) be also a normalized pr function of the kind

$$\Phi(s) = \frac{\beta(s)}{s\alpha(s)} = \frac{s^m + \beta_{m-1}s^{m-1} + \cdots + \beta_1 s + \beta_0}{s\left(s^m + \alpha_{m-1}s^{m-1} + \cdots + \alpha_1 s + \alpha_0\right)} \quad . \tag{8}$$

The functions $\varphi(s)$ and $\Phi(s)$ are bi-order-m functions; m is an even or odd integer. Substituting Eqs. (2), ..., (8) in Eq. (1), we obtain

$$F(s) = \frac{s^2 a(s)\alpha(s) + x(\underline{n} - 1)^2\, s a(s)\beta(s) + x b(s)\beta(s)/v\underline{n}}{s^2 a(s)\alpha(s) + s b(s)\,\alpha(s)/v\underline{n} + x\underline{n} b(s)\beta(s)/v} \quad . \tag{9}$$

Evidently F is a normalized bi-order-n function; it is pr and the order n of its numerator and denominator polynomials is

$$n = 2m + 2 \quad . \tag{10}$$

The function can be written in the form

$$F(s) = \frac{\bar{N}(s)}{\bar{D}(s)} = \frac{s^n + \bar{N}_{n-1}s^{n-1} + \cdots + \bar{N}_1 s + \bar{N}_0}{s^n + \bar{D}_{n-1}s^{n-1} + \cdots + \bar{D}_1 s + \bar{D}_0} \quad . \tag{11}$$

The bars over the capital letters in Eq. (11) indicate that $F(s)$ is a bi-order-n function that can be obtained in the foregoing manner. In other words, $F(s)$ can be decomposed into a circuit according to Figure 1 when it is considered as an impedance function, or again into a circuit according to Figure 2 when it is considered as an admittance function. Not any pr bi-order-n function can be decomposed in this way, but some can be prepared at least for decomposition in this way.

Let us consider a trivial example: Let $m = 0$. Then $\varphi(s) = 1/\phi(s) = s$ and the function $F(s)$ in this event according to Eq. (9) becomes biquadratic. Let us denote in this particular event

$$F'(\varepsilon) = \frac{s^2 + \bar{N}'_1 s + \bar{N}'_0}{s^2 + \bar{D}'_1 s + \bar{D}'_0} \quad . \tag{12}$$

The coefficients in the numerator and the denominator of $F'(s)$ are (Haase, 1966)

$$\bar{N}'_1 = x(\underline{n} - 1)^2 \quad , \tag{13a}$$

$$\bar{D}'_1 = 1/v\underline{n} \quad , \tag{13b}$$

$$\bar{N}'_0 = x/v\underline{n} \quad , \tag{13c}$$

$$\bar{D}'_0 = x\underline{n}/v \quad . \tag{13d}$$

Inversely

$$v = \frac{1}{D'_1}\sqrt{\frac{N'_0}{D'_0}} \quad , \tag{14a}$$

$$x = \frac{N'_0}{D'_1} \quad , \tag{14b}$$

$$\underline{n} = \sqrt{\frac{D'_0}{N'_0}} \quad , \tag{14c}$$

$$\underline{n} - 1 = \sqrt{\frac{N'_1\, D'_1}{N'_0}} \tag{14d}$$

Since we assume that $\underline{n} > 1$, $D'_0 > N'_0$. Necessarily

$$\sqrt{N'_1\, D'_1} = \left(\sqrt{D'_0} - \sqrt{N'_0}\right)^2 \tag{15}$$

It is well known that a biquadratic function

$$F(s) = \frac{s^2 + N_1 s + N_0}{s^2 + D_1 s + D_0} \tag{16}$$

is pr if

$$N_1 D_1 \geq \left(\sqrt{D_0} - \sqrt{N_0}\right)^2 \quad . \tag{17}$$

Incidentally, Eq. (15) represents the lower bound of Eq. (17). A biquadratic impedance function $F'(s)$ can be decomposed according to Figure 1 if Eq. (15) holds. The circuit in this event is the well-known Brune circuit where $u\varphi(s)$, $v\varphi(s)$, and $w\varphi(s)$ are inductive impedances and $x\phi(s)$ is a capacitive impedance.

Next let us consider a less trivial example: Let $m = 1$. Then (for the sake of brevity omitting the subindex " 0 ")

$$\varphi(s) = s\,\frac{s+a}{s+b} \text{ with } a > b \text{ and both positive} \quad , \tag{18}$$

$$\phi(s) = \frac{s+\beta}{s(s+\alpha)} \text{ with } \alpha > \beta \text{ and both positive} \quad . \tag{19}$$

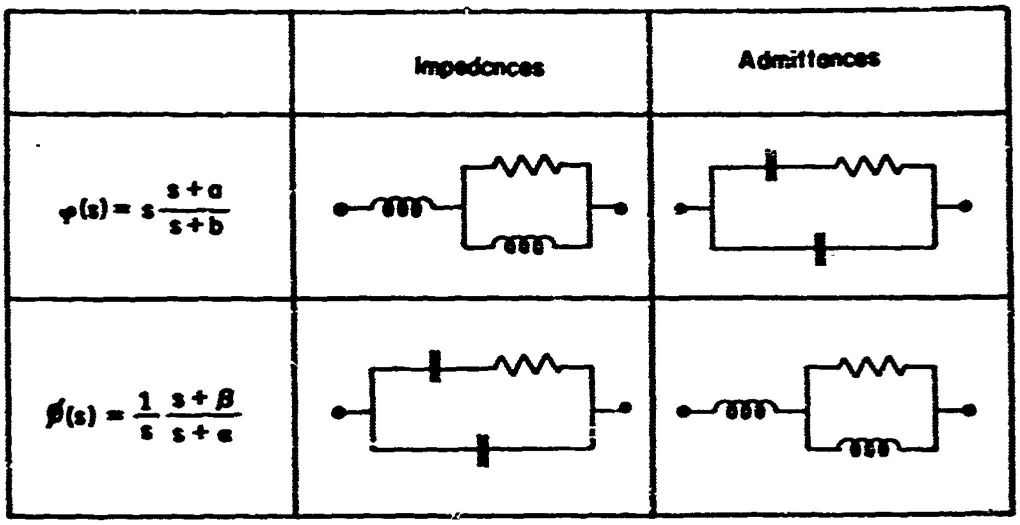

Figure 3. Circuits Representing $\varphi(s)$ and $\phi(s)$

Circuits realizing $\varphi(s)$ and $\phi(s)$ in the impedance and the admittance interpretation are shown in Figure 3. The function $\bar{F}(s)$ in this event is biquartic according to Eq. (9); it can be written as

$$\bar{F}(s) = \frac{s^4 + \bar{N}_3 s^3 + \bar{N}_2 s^2 + \bar{N}_1 s + \bar{N}_0}{s^4 + \bar{D}_3 s^3 + \bar{D}_2 s^2 + \bar{D}_1 s + \bar{D}_0} \quad , \tag{20}$$

and by comparison between Eq. (20) and Eq. (9) and with Eqs. (14a, ..., d)

$$\bar{N}_3 = (a + \alpha) + \bar{N}'_1 \quad , \tag{21a}$$

$$\bar{N}_2 = a\alpha + \bar{N}'_1(a + \beta) + \bar{N}'_0 \quad , \tag{21b}$$

$$\bar{N}_1 = \bar{N}'_1 a\beta + \bar{N}'_0(b + \beta) \quad , \tag{21c}$$

$$\bar{N}_0 = \bar{N}'_0 b\beta \quad , \tag{21d}$$

$$\bar{D}_3 = (a + \alpha) + \bar{D}'_1 \quad , \tag{22a}$$

$$\bar{D}_2 = a\alpha + \bar{D}'_1(b + \alpha) + \bar{D}'_0 \quad . \tag{22b}$$

$$\bar{D}_1 = \bar{D}'_1 b\alpha + \bar{D}'_0(b + \beta) \quad , \tag{22c}$$

$$\bar{D}_0 = \bar{D}'_0 b\beta \quad . \tag{22d}$$

As a numerical example let

$\bar{N}_3 = 3.75$	,	$\bar{D}_3 = 1.728571$	,
$\bar{N}_2 = 3.949286$	,	$\bar{D}_2 = 3.634285$	,
$\bar{N}_1 = 1.271429$	,	$\bar{D}_1 = 2.32$	,
$\bar{N}_0 = 0.137143$	,	$\bar{D}_0 = 0.42$	.

The numerical values of the coefficients $\bar{N}_i$ and $\bar{D}_i$ $(0 \leq i \leq 3)$ are such that $\bar{F}(s)$ can be decomposed in the attempted way; we shall use this example to show the decomposition in a more concise way than the algebraic formulation. Our particular problem is to determine N'_1, N'_0, D'_1, D'_0, a, b, α, and β by Eqs. (21a, ..., d) and Eqs. (22a, ..., d). Although more cumbersome, a bisextic function resulting from $m = 2$ and so forth can be decomposed by way of a similar procedure.

The circuit in Figure 1 has two extraordinary driving conditions. Whenever the shunt impedance $V + X$ is zero, we measure at the input the driving impedance $\bar{F}(s) = u\varphi(s)$. This is also true whenever the impedance $W + Z$ is zero. Let s_1 denote the set of solutions of the equation

$$v\left[\varphi(s) + \frac{x}{v}\phi(s)\right] = 0 \quad , \tag{23}$$

and let s_2 denote the set of solutions of the equation

$$w\left[\varphi(s) + \frac{z}{w}\right] = 0 \quad . \tag{24}$$

In our particular example Eq. (23) becomes

$$s^2(s + a)(s + \alpha) + \frac{x}{v}(s + b)(s + \beta) = 0 \quad , \tag{25}$$

which is of the order four, and Eq. (24) becomes

$$s(s + a) + \frac{z}{w}(s + b) = 0 \quad , \tag{26}$$

which is quadratic. Note that since w is a negative constant, Eq. (26) has both a positive solution $s = s_{21}$ and a negative solution $s = -s_{22}$.

With the set of zeros s_2

$$F(s_2) = u\varphi(s_2) = v(\underline{n} - 1)\varphi(s_2) \quad . \tag{27}$$

Note that by the definition of w in Eq. (2c) and by Eq. (24)

$$\varphi(s_2) = z/w = z\underline{n}/v(\underline{n} - 1) \quad , \tag{28}$$

and by

$$F(0) = F_0/D_0 = 1/\underline{n}^2 \tag{29}$$

$$F(\infty) = 1 = z\underline{n}^2 \tag{30}$$

the constants z and $\underline{n}$ are known. Hence Eq. (27) becomes

$$A(s) = s^4 + A_3s^3 + A_2s^2 + A_1s + A_0 = 0 \quad , \tag{31}$$

where

$$A_i = \frac{N_i\sqrt{D_0} - D_i\sqrt{N_0}}{\sqrt{D_0} - \sqrt{N_0}} \qquad (0 \leq i \leq 3) \tag{31a}$$

The coefficients A_i of Eq. (31) are thus known and the equation can be solved. It is of the order four, with four solutions, two of which are the solutions s_{21} and $-s_{22}$ of Eq. (24). Therefore Eq. (31) is redundant. Generally it offers twice the number of solutions as Eq. (24). It can be shown (Haase, 1963) that if $F(s)$ is biquartic, Eq. (31) has real roots throughout. One root is positive; the others are negative. Only the positive root $s = s_{21}$ can be identified as belonging to set s_2. One of the negative roots is $s = -s_{22}$ and the redundant roots are $s = s_{r1}$ and $s = s_{r2}$. Let us now define

$$p_2 = -(s_{21} - s_{22}) \quad , \tag{32a}$$

$$q_2 = s_{21}s_{22} \quad , \tag{32b}$$

$$p_r = -(s_{r1} + s_{r2}) \quad , \tag{33a}$$

$$q_r = s_{r1} s_{r2} \quad . \tag{33b}$$

Then Eq. (31) becomes

$$A(s) = (s^2 + sp_2 - q_2)(s^2 + sp_r + q_r) = 0 \quad . \tag{33}$$

Thus far we are unable to identify p_2, q_2, p_r, and q_r and in our example where according to Eq. (31a)

$$A_3 = 6.445242 \,,\ A_2 = 4.369286 \,,\ A_1 = -0.126667 \,,\ A_0 = -0.24$$

and

$$A(s) = (s - 0.214654)(s + 5.672338)(s + 0.277664)(s + 0.709891)$$

we have three choices:

(1) $p_2 = 5.457684$, $\quad p_r = 0.987555$,
$q_2 = 1.217590$, $\quad q_r = 0.197111$,

(2) $p_2 = 0.063010$, $\quad p_r = 6.382229$,
$q_2 = 0.059602$, $\quad q_r = 4.026742$,

(3) $p_2 = 0.495237$, $\quad p_r = 5.950002$,
$q_2 = 0.152381$, $\quad q_r = 1.575004$.

One of these choices is the correct one. By some algebraic manipulations we are able to derive from Eqs. (21) and (22)

$$(p_2 - p_r)\bar{N}'_1 = p_r(\bar{D}_3 - \bar{N}_3) - (\bar{D}_2 - \bar{N}_2) + q_2(\underline{n} - 1) - q_r(1 - 1/\underline{n}) \quad , \tag{34a}$$

$$\bar{D}'_1 = \bar{N}'_1 + (\bar{D}_3 - \bar{N}_3) \quad , \tag{34b}$$

so that for each of the choices $\bar{N}'_1$ and $\bar{D}'_1$ can be determined. Further, since by Eq. (21d) and Eq. (22d)

$$\bar{D}_0 - \bar{N}_0 = b\beta(\bar{D}'_0 - \bar{N}'_0) \quad , \tag{35a}$$

and by Eq. (15)

$$\sqrt{\bar{N}'_1 \bar{D}'_1} = \left(\sqrt{\bar{D}'_0} - \sqrt{\bar{N}'_0}\right)^2 ; \tag{35b}$$

also D'_0 and N'_0 can be determined for each choice. It can also be shown that

$$a = p_2 + \sqrt{\bar{N}'_0 \bar{D}'_1 / \bar{N}'_1} \; , \tag{36a}$$

$$b = q_2 \sqrt{\bar{N}'_1 / \bar{N}'_0 \bar{D}'_1} \; , \tag{36b}$$

$$\alpha = p_r - \sqrt{\bar{D}'_0 \bar{N}'_1 / \bar{D}'_1} \; , \tag{37a}$$

$$\beta = q_r \sqrt{\bar{D}'_1 / \bar{D}'_0 \bar{N}'_1} \; . \tag{37b}$$

Only the correct choice yields all coefficients $\bar{N}'_i$ and $\bar{D}'_i$ positive and positive constants $a > b$ and $\alpha > \beta$. In our numerical example, choice (3) is the correct one according to this discrimination: it yields

$\bar{N}'_1 = 2.25$, $\bar{N}'_0 = 0.914286$, $\bar{D}'_1 = 0.228571$, $\bar{D}'_0 = 2.8$, $a = 0.8$, $b = 0.5$,
$\alpha = 0.7$, $\beta = 0.3$.

The last condition that we can derive from Eqs. (22) and (23) is that with the correct choice

$$(\bar{D}_3 - \bar{N}_3)c_3 + (\bar{D}_2 - \bar{N}_2)c_2 + (\bar{D}_1 - \bar{N}_1)c_1 + (\bar{D}_0 - \bar{N}_0)c_0 = 0 \; , \tag{38}$$

when

$$c_3 = p_2 q_r + p_r q_2 \; , \tag{38a}$$

$$c_2 = -(q_2 + q_r) \; , \tag{38b}$$

$$c_1 = -(p_2 - p_r) \; , \tag{38c}$$

$$c_0 = -\left[\frac{p_r c_1 + c_2}{q_r}\,\underline{n} - \frac{p_2 c_1 - c_2}{q_2}\right] \frac{1}{1 + \underline{n}} \; . \tag{38d}$$

Our numerical example shows that Eq. (38) is true.

If $F(s) = F'(s)$ is biquadratic, the function can be decomposed in the attempted way if Eq. (15) holds. Unfortunately, we do not have such a simple test for the bi-quartic function or a function of a higher bi-order-n. As we have shown, we are able to make the correct choice, but this alone does not ensure that the function can be decomposed as attempted. We show this in the following numerical example. Assume a biquartic impedance function $F(s) = N(s)/D(s)$ where

$N_3 = 3.413095$, $\qquad D_3 = 1.728571$,

$N_2 = 3.896786$, $\qquad D_2 = 3.634285$,

$N_1 = 1.446191$, $\qquad D_1 = 2.32$,

$N_0 = 0.184286$, $\qquad D_0 = 0.42$.

Then $\underline{n}^2 = 1.509658^2$. For this example

$$A(s) = (s - 0.237544)(s + 5.973491)(s + 0.278596)(s + 0.703759)$$

and the correct choice turns out to be

$p_2 = 0.466215$, $\qquad p_r = 6.252087$,

$q_2 = 0.167174$, $\qquad q_r = 1.664191$.

But instead of yielding 0 , we obtain by Eq. (38) + 0.029445. This shows that $F(s)$ in this example cannot be decomposed as desired. For this reason let us subtract a positive constant N_{m1} from $F(s)$. By choosing $N_{m1} = 0.2$, we find that the function $\tilde{F}(s) = |F(s) - N_{m1}|/(1 - N_{m1})$ is normalized and still pr . It has the following coefficients

$\tilde{N}_3 = 3.834226$, $\qquad \tilde{D}_3 = 1.728571$,

$\tilde{N}_2 = 3.962411$, $\qquad \tilde{D}_2 = 3.634285$,

$\tilde{N}_1 = 1.227739$, $\qquad \tilde{D}_1 = 2.32$

$\tilde{N}_0 = 0.125358$, $\qquad \tilde{D}_0 = 0.42$.

Then $\tilde{\underline{n}}^2 = 1.830411^2$. Solving Eq. (31) and making the correct choice, we obtain

$p_2 = 0.503889$, $\qquad p_r = 5.666015$,

$q_2 = 0.148057$, $\qquad q_r = 1.549788$.

Equation (38) yields -0.011711 instead of 0. But since the result obtained for $F(s)$ was positive, we know now that we are able to find a constant $N_m < 0.2$ that yields exactly 0 by Eq. (38). In fact, if we choose $N_m = 1/6$, then $\tilde{F}(s)$ has the coefficients of the first numerical example.

We have shown that we have been able to prepare the function $F(s)$ for decomposition by subtracting a constant. However, this is not the only possibility. Assume for instance that the denominator of a function $F(s)$ is $D(s) = s^4 + D_3 s^3 + \cdots + D_0 = (s^2 + d_{11}s + d_{10})(s^2 + d_{21}s + d_{20})$ and $d_{11}^2 < 4d_{10}$, $d_{21}^2 < 4d_{20}$. In this event we can subtract either immittance

$$\frac{n_{11}s + n_{10}}{s^2 + d_{11}s + d_{10}} \quad \text{or} \quad \frac{n_{21}s + n_{20}}{s^2 + d_{21}s + d_{20}}$$

and by the subtraction, the difference function is still a bi-order-4 function and with a fairly wide choice of the constants n_{11}, n_{21}, n_{10}, n_{20} the difference function can be retained to be pr. However, decomposition may not be possible by subtracting only a constant.

The realization of the circuit in Figure 1 requires a negative impedance converter, since W is a negative impedance. Such a converter, however, may bring up technical difficulties since the rank of W is $2m + 1$ according to Eq. (2c) and Eq. (7). But we have shown (Haase, 1966) that the circuit in Figure 1 is equivalent in its driving-point impedance with the circuit in Figure 4. This circuit is terminated by the positive impedance $z'\varphi(s)$, part of its shunt branch is the positive impedance $x'\phi(s)$, and it implies a resistance star composed of the resistances R'_u, R'_v, and R'_w where

$$1/R'_u + 1/R'_v + 1/R'_w = 0 \tag{39}$$

and

$$R'_u = R'_v(\underline{n}' - 1) \quad , \tag{40a}$$

$$R'_w = -R'_v(\underline{n}' - 1)/n' \quad . \tag{40b}$$

According to transformation formulas given by Haase (1966)

$$x' = x(\underline{n} - 1)^2 \quad , \tag{41}$$

$$z' = v(\underline{n} - 1)^2/n \quad , \tag{42}$$

$$\underline{n}' = \underline{n}/(\underline{n} - 1) \quad , \tag{43}$$

$$R'_v = 1/\underline{n}' \quad . \tag{44}$$

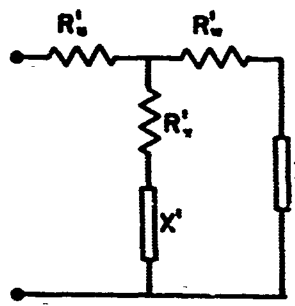

Figure 4. Transformed Circuit of Figure 1

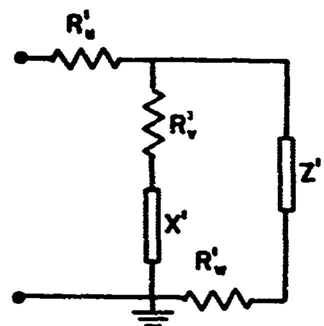

Figure 5. Circuit as in Figure 4 With Resistance R'_w Grounded

Instead of a negative impedance of the rank $2m + 1$ the circuit in Figure 4 contains a negative resistance as the only negative element. This resistance can be realized by a tunnel diode for instance. Since the elements in the circuit can be rearranged, it is possible to put the tunnel diode in the base branch where it can be grounded as shown in Figure 5.

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